MA/MSCMT - 10

June - Examination 2016

M.A./M.Sc. (Final) Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT - 10

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define supporting Hyper plane.

(ii) Given a problem min $f(x, y, z) = dyz$ subject to $x + y + z = 15$; $2x - y + 2z = 20$; $x, y, z \geq 0$; how many Lagrange multiplier are required to solve this problem.

(iii) Write necessary condition for the function $f(x, \lambda)$ have a saddle point on (x_o, λ_o).

(iv) Write Hessian matrix for $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$.

(v) Test the definiteness of the quadratic form

$$X^T A X = (x_1, x_2, x_3) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(vi) Define feasible point for the Dual.

(vii) Write Dual of Max $f(X) = C^T X + \frac{1}{2} X^T G X$ subject to $AX = b;\ X \geq 0$

(viii) Define 'stage' and 'state' in Dynamic Programming.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Show that if $f(x)$ is continuous, $f(x) \geq 0 \; -\infty < x < \infty$ then the function $\phi(x) = \int_x^\infty (y - x)\, f(y)\, dy$ is a convex function provided the integral converges.

3) Obtain a set of necessary condition for the non-linear programming problem: maximize $z = x_1^2 + 3x_2^2 + 5x_3^2$

s.t. $5x_1 + 2x_2 + x_3 = 5$

$x_1, x_2, x_3 \geq 0$

4) A positive quantity b is divided into n parts in such a way that the product of the n parts is to be maximum. Use Lagrange multiplier technique, obtain the optimal sub division.

5) Determine the optimal solution of the following non-linear programming problem; using the Kuhn - Tucker conditions:

minimize $f(x_1, x_2) = x_1^2 + 2x_2^2 - x_1 x_2$

subject to $x_1 + x_2 \geq 8$

$x_1, x_2 \geq 0$

6) Solve the following quadratic programming problem by Beale's Method:

maximize $f(x_1, x_2) = 2x_1 + 3x_2 - 2x_1^2$

s.t. $x_1 + 4x_2 \leq 4$

$x_1 + 2x_2 \leq 2$

$x_1 , x_2 \geq 0$

7) Make use of Dynamic Programming, show that

$\sum_{i=1}^{n} pi \log pi$ subject to $\sum_{i=1}^{n} pi = 1, \ pi > 0$

is minimum when $p_1 = p_2 = \ldots\ldots = p_n = \frac{1}{n}$

8) Solve by Dynamic Programming

max $Z = x_1 + 9x_2$

s.t. $2x_1 + x_2 \leq 25; \quad x_2 \leq 11$

and $x_1 \geq 0, \ x_2 \leq 0$

9) Derive the Dual Function of non-linear programming problem:

maximize $f(x)$

s.t. $gi(x) \geq 0 \quad i = 1, 2, \ldots\ldots, m$

$hj(x) = 0 \quad j = 1, 2, \ldots\ldots, p$

Section - C $\quad$ **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain (04) Long Answer Type Questions. Examinees will have to answer any **Two** (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Solve the following LPP

max $Z = 3x_1 + 5x_2 + 2x_3$

s.t. $x_1 + 2x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 23$

and $0 \leq x_1 \leq 4$; $0 \leq x_2 \leq 5$; $0 \leq x_3 \leq 3$

$x_2 \geq 2$

11) Use Branch and Bound Method, Solve the following LPP:

minimize $Z = 4x_1 + 3x_2$

s.t. $5x_1 + 3x_2 \geq 30$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1, x_2 \geq 0$ are integers.

12) Solve the following quadratic programming problem using Walfe's method.

min $f\left(x_1, x_2\right) = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - x_2$

s.t. $2x_1 + x_2 \leq 1$

$x_1, x_2 \geq 0$

13) Solve the following convex seperable programming problem:

min $Z = x_1^2 - 2x_1 - x_2$

Such that $2x_1^2 + 3x_2^2 \leq 6$

and $x_1, x_2 \geq 0$